

Elementary Excitations of a Relativistic Scalar Plasma System

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ABSTRACT

We investigate the physics of elementary excitations for the so called relativistic scalar plasma system. Following the standard many-body procedure we have obtained the RPA equations for this model by linearizing the TDHFB equations of motion around equilibrium and shown that these oscillation modes give one-meson and two-fermion state of the theory. The resulting equations have a closed solution, from which one can examine the spectrum of excitation modes. In particular, our results indicate existence of bound state for certain region of phase diagram.

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I. INTRODUCTION

In a previous work [1] (hereafter referred to as I) we have presented a framework to investigate the initial-value problem in the context of interacting fermion-scalar field theories. The method allows one to obtain a set of self-consistent equations for the expectation values of linear and bilinear forms of field operators. These dynamical equations acquire the structure of kinetic type, where the lowest order of the approximation corresponds to the usual gaussian mean-field approximation (collisionless). As application, we have implemented a zero-order calculation within the simplest context of relativistic scalar plasma system. We have shown that the usual prescription of renormalization can also apply to these nonperturbative calculation. In particular, we have obtained a finite expression for the energy density and the numerical results suggested that the system presents always a single stable minimum .

In continuation of I we will report in this paper a particular application of the renormalized nonlinear obtained the previous publication. We follow here a recent work by Kerman and Lin [2, 3] in order to investigate the near equilibrium dynamics around the stationary solution. We shall show that one-meson and two-(quasi)fermion physics can be studied from the linear approximation of the mean-field equations. In particular, one can solve these equations in a closed form and find scattering amplitude as well as the conditions for the two-fermion bound state.

For completeness and notational purpose, we repeat here a few key equations of I. A summary of derivation for these equations is shown in Appendix A. For the scalar plasma model, the dynamics are governed by the hamiltonian

$$\begin{aligned}
 H &= \int_{\mathbf{x}} \mathcal{H} , \\
 \mathcal{H} &= -\bar{\psi}(i\vec{\gamma} \cdot \vec{\partial} - m)\psi - g\bar{\psi}\phi\psi + \frac{1}{8\pi} \left[\frac{(4\pi)^2}{1+Z} \Pi^2 + (1+Z)|\partial\phi|^2 + \mu^2\phi^2 \right] + \mathcal{H}_c ,
 \end{aligned}
 \tag{1.1}$$

[We use the notation: $\int_{\mathbf{x}} = \int d^3x$] where ψ is a spin- $\frac{1}{2}$ field while ϕ is a scalar field. The

parameters m and μ are, respectively, the mass of fermion and scalar particles and g is the coupling constant. The last term of this expression is the counterterms necessary to remove the infinities occurring later [1, 4],

$$4\pi\mathcal{H}_c = \frac{A}{1!}\phi + \frac{\delta\mu^2}{2!}\phi^2 + \frac{C}{3!}\phi^3 + \frac{D}{4!}\phi^4 \quad , \quad (1.2)$$

where the coefficients A , $\delta\mu^2$, C , D , and Z are infinity constants to be defined later.

In order to study the dynamics of the system, we focus on the selected set of observables, which are related to the expectation values of linear and bilinear forms of field operators, referred to as gaussian variables. The time evolution of these quantities obeys the Heisenberg equation of motion,

$$i\langle\dot{\mathcal{O}}\rangle = Tr_{\text{BF}}[\mathcal{O}, H]\mathcal{F} \quad (1.3)$$

where \mathcal{O} could be $\phi(x)$, $\phi(x)\phi(x)$, $\bar{\psi}(x)\psi(x)$ and etc., and F is the state of the system in the Heisenberg picture. As approximation we replace the full density \mathcal{F} by a truncated ansatz $\mathcal{F}_0(t)$. By construction, \mathcal{F}_0 reproduces the corresponding \mathcal{F} averages for linear or bilinear field operators [see Eqs.(41) of [1]]. In particular, we have used a formulation appropriate for the many-body problem, so that \mathcal{F}_0 can be written in the momentum basis as [5, 6]

$$\begin{aligned} \mathcal{F}_0 &= \mathcal{F}_0^{\text{B}}\mathcal{F}_0^{\text{F}} \\ \mathcal{F}_0^{\text{B}} &= \prod_{\mathbf{p}} \frac{1}{1 + \nu_{\mathbf{p}}} \left(\frac{\nu_{\mathbf{p}}}{1 + \nu_{\mathbf{p}}} \right)^{\beta_{\mathbf{p}}^{\dagger}\beta_{\mathbf{p}}} \end{aligned} \quad (1.4)$$

$$\mathcal{F}_0^{\text{F}} = \prod_{\mathbf{k},s,\lambda} [\nu_{\mathbf{k},s}^{(\lambda)}\alpha_{\mathbf{k},s}^{(\lambda)\dagger}\alpha_{\mathbf{k},s}^{(\lambda)} + (1 - \nu_{\mathbf{k},s}^{(\lambda)})\alpha_{\mathbf{k},s}^{(\lambda)}\alpha_{\mathbf{k},s}^{(\lambda)\dagger}] \quad , \quad (1.5)$$

where α (α^{\dagger}) and β (β^{\dagger}) stand for Bogoliubov quasi-particle annihilation (creation) operators for fermion and boson respectively. In doing so, the gaussian variables are now represented by the Bogoliubov parameters [see (A.3)-(A.5)] and its equations of motion, can be obtained directly using (1.1)-(1.5). The resulting expressions are:

$$\dot{\varphi}_{\mathbf{k}} = g\langle\phi\rangle\frac{|\mathbf{k}|}{k_0}\sin\gamma_{\mathbf{k}} \quad (1.6)$$

$$\dot{\gamma}_{\mathbf{k}} = 2k_0 - 2gm\langle\phi\rangle\frac{1}{k_0}\left(1 - \frac{|\mathbf{k}|}{m}\cot 2\varphi_{\mathbf{k}}\cos\gamma_{\mathbf{k}}\right) \quad (1.7)$$

$$\langle\dot{\phi}\rangle = \frac{4\pi}{(1+Z)}\langle\Pi\rangle \quad (1.8)$$

$$\begin{aligned} \langle\dot{\Pi}\rangle &= -\left(\frac{A}{4\pi} + \frac{C}{2}G(\mu)\right) - \left(\frac{\mu^2}{4\pi} + \frac{\delta\mu^2}{4\pi} + \frac{D}{2}G(\mu)\right)\langle\phi\rangle \\ &\quad - \frac{C}{8\pi}\langle\phi\rangle^2 - \frac{D}{24\pi}\langle\phi\rangle^3 - \frac{2g}{(2\pi)^3}[I_1(m) + I_2(m)] \end{aligned} \quad (1.9)$$

where

$$G(\mu) = \frac{1}{(2\pi)^3} \int d^3\mathbf{p} \frac{1}{2(\mathbf{p}^2 + \mu^2)^{1/2}} \quad (1.10)$$

$$I_1(m) = \int d^3\mathbf{k} \frac{m}{(\mathbf{k}^2 + m^2)^{1/2}} \cos 2\varphi_{\mathbf{k}} \quad (1.11)$$

$$I_2(m) = \int d^3\mathbf{k} \frac{|\mathbf{k}|}{(\mathbf{k}^2 + m^2)^{1/2}} \sin 2\varphi_{\mathbf{k}} \cos \gamma_{\mathbf{k}} \quad (1.12)$$

The mean-field energy density, on the other hand, reads as

$$\begin{aligned} \frac{\langle H \rangle}{V} &= \frac{1}{V} \text{Tr} H \mathcal{F}_0 \\ &= -2 \int_{\mathbf{k}} k_0 \cos 2\varphi_{\mathbf{k}} + 2g\langle\phi\rangle [I_1(m) + I_2(m)] \end{aligned}$$

$$\begin{aligned}
& + \frac{1}{4\pi} \left(\frac{\mu^2}{2} \langle \phi \rangle^2 + \frac{\langle \Pi \rangle^2}{2} \right) + \left(\frac{A}{4\pi} + \frac{C}{2} G(\mu) \right) \langle \phi \rangle \\
& + \left(\frac{\mu^2}{8\pi} + \frac{\delta\mu^2}{8\pi} + \frac{D}{4} G(\mu) \right) \langle \phi \rangle^2 + \frac{C}{24\pi} \langle \phi \rangle^3 + \frac{D}{96\pi} \langle \phi \rangle^4
\end{aligned} \tag{1.13}$$

The equations above describe the real-time evolution for the relativistic scalar plasma system in the gaussian mean-field approximation. They are nonlinear time-dependent field equations. Therefore, a closed solution is not easily constructed. Here, we will consider the equations in the equilibrium situation and the small oscillation regime. In these cases, a closed solution can be found allowing us to examine diverse properties of the theory. The structure of this paper is as follows. In Sec. II we shall derive the RPA equation for this model by considering near equilibrium dynamics about the equilibrium solutions obtained in I. Section III shows an analytical solutions for these RPA equations using a well know procedure of the scattering theory. In Section IV we discuss the question of renormalization within the context of scattering amplitudes and examine the possibility of existence of bound state solutions.

II. NEAR EQUILIBRIUM DYNAMICS

As a function of the Bogoliubov parameters, the energy density is a function of these variables. A minimum of (1.13) corresponds to the ground state of the system and small amplitude motion about the minima is described by the linearized approximation of the gaussian equations motion (1.6)-(1.9), yielding a set of harmonic oscillators [7]. The normal modes and the eigenvectors are the RPA solutions. In the field theoretical context these can be seen as the energy and the wavefunctions of quantum particles. In this section we derive the RPA equations of the model and next section will discussion solutions for this problem.

Let us consider first the static problem (see section VI-b of I). Recalling (1.6)-(1.9), the stationary conditions require $\dot{\gamma}_k = \dot{\varphi}_k = \dot{\kappa}_p = \dot{\eta}_p = \langle \dot{\phi} \rangle = \langle \dot{\Pi} \rangle = 0$. In Section IV-b of I we have discussed in detail the renormalization conditions and the solutions and for this set of equations. In particular, the following choice for the coefficients of \mathcal{H}_c satisfies the self-consistency condition [1, 4]

$$D = \pm 48\pi g^4 L(m) , \quad (2.1)$$

$$\delta\mu^2 = \mp 24\pi^2 g^4 L(m) G(\mu) \mp 16\pi g^2 G(0) \pm 24\pi m^2 g^2 L(m) , \quad (2.2)$$

$$C = \mp 48\pi m g^3 L(m) , \quad (2.3)$$

$$A = \pm 24\pi m g^3 L(m) G(\mu) \pm 16\pi m g G(m) , \quad (2.4)$$

with

$$L(m) \equiv \int_{\mathbf{k}} \frac{1}{2\mathbf{k}^2(\mathbf{k}^2 + m^2)^{1/2}} . \quad (2.5)$$

In doing so, the finite static equations of the system can be regrouped as follows:

$$\sin \gamma_{\mathbf{k}}|_{\text{eq}} = 0 \quad (2.6)$$

$$\cot 2\varphi_{\mathbf{k}}|_{\text{eq}} = -\frac{(\mathbf{k}^2 + \bar{m}m)}{|\mathbf{k}|(m - \bar{m})} \quad (2.7)$$

$$\langle \Pi \rangle|_{\text{eq}} = 0 \quad (2.8)$$

$$\frac{\pi}{2}\mu^2 \langle \phi \rangle|_{\text{eq}} - g\bar{m}^3 \left[\ln \left(\frac{\bar{m}}{m} \right) + \frac{1}{2} \right] = 0 . \quad (2.9)$$

where \bar{m} is the effective fermion mass,

$$\bar{m} \equiv m - g\langle \phi \rangle|_{\text{eq}} . \quad (2.10)$$

The equation (2.6)-(2.9) can be solved numerically for any given values of μ and g in unit of m .

Investigation of the near equilibrium motion proceeds by considering flutuactions around the stationary point

$$\begin{aligned}
\varphi_{\mathbf{k}} &= \varphi_{\mathbf{k}}^{(0)} + \delta\varphi_{\mathbf{k}} \\
\gamma_{\mathbf{k}} &= \gamma_{\mathbf{k}}^{(0)} + \delta\gamma_{\mathbf{k}} \\
\langle\phi\rangle &= \langle\phi\rangle^{(0)} + \delta\langle\phi\rangle \\
\langle\Pi\rangle &= \langle\Pi\rangle^{(0)} + \delta\langle\Pi\rangle \ ,
\end{aligned} \tag{2.11}$$

where $\varphi_{\mathbf{k}}^{(0)}, \gamma_{\mathbf{k}}^{(0)}, \langle\phi\rangle^{(0)}$ and $\langle\Pi\rangle^{(0)}$ satisfy (2.6)-(2.9) and the quantities $\delta\varphi_{\mathbf{k}}, \delta\gamma_{\mathbf{k}}, \delta\langle\phi\rangle|_{\text{eq}}$ and $\delta\langle\Pi\rangle|_{\text{eq}}$ will be assumed small in our approximation. Next step is to expand (1.6)-(1.9) up to first order in these fluctuations to yield

$$\delta\dot{\varphi}_{\mathbf{k}} = g\langle\phi\rangle^{(0)}\frac{|\mathbf{k}|}{k_0}\delta\gamma_{\mathbf{k}} \tag{2.12}$$

$$g\langle\phi\rangle^{(0)}|\mathbf{k}|\delta\dot{\gamma}_{\mathbf{k}} = -4k_0(\mathbf{k}^2 + M^2)\delta\varphi_{\mathbf{k}} - 2g|\mathbf{k}|k_0\delta\langle\phi\rangle \tag{2.13}$$

$$\delta\langle\dot{\phi}\rangle = \frac{4\pi}{(1+Z)}\delta\langle\Pi\rangle \tag{2.14}$$

$$\begin{aligned}
\delta\langle\dot{\Pi}\rangle &= -\left(\frac{\mu^2}{4\pi} + \frac{\delta\mu^2}{4\pi} + \frac{D}{2}G(\mu)\right)\delta\langle\phi\rangle - \frac{C}{4\pi}\langle\phi\rangle^{(0)}\delta\langle\phi\rangle \\
&\quad - \frac{D}{8\pi}\langle\phi\rangle^{(0)2}\delta\langle\phi\rangle + \frac{4g}{(2\pi)^3}\int d^3\mathbf{k}'\frac{|\mathbf{k}'|}{(\mathbf{k}'^2 + M^2)^{1/2}}\delta\varphi_{\mathbf{k}} \ .
\end{aligned} \tag{2.15}$$

Eliminating the momenta $\delta\gamma_{\mathbf{k}}$ and $\delta\langle\Pi\rangle$ we might rewrite (2.12)-(2.15) in a more compact second order equations:

$$\delta\ddot{\varphi}_{\mathbf{k}} = -4\bar{\omega}^2\delta\varphi_{\mathbf{k}} - 2g|\mathbf{k}|\delta\langle\phi\rangle \tag{2.16}$$

$$(1 + Z)\delta\langle\ddot{\phi}\rangle = -(\mu^2 + \Sigma)\delta\langle\phi\rangle + 16\pi g \int_{\mathbf{k}} h(\mathbf{k})\delta\varphi_{\mathbf{k}} \quad (2.17)$$

where we use notations

$$h(\mathbf{k}) = \frac{|\mathbf{k}|}{\bar{k}_0} \quad (2.18)$$

with $\bar{k}_0 = \sqrt{\mathbf{k}^2 + \bar{m}^2}$ and

$$\Sigma \equiv \delta\mu^2 + \frac{D}{2}4\pi G(\mu) + C\langle\phi\rangle^{(0)} + \frac{D}{2}\langle\phi\rangle^{(0)^2} . \quad (2.19)$$

Thus, the small oscillation dynamics is described by coupled linear oscillator equations as usual in RPA treatment. In particular, these modes decouple when $g = 0$ yielding two simple oscillator equations.

A solution to this problem involves determining the normal modes of small oscillation and their frequencies. On the other hand, earlier studies have been demonstrated that these elementary excitations can be interpreted as quantum particles. In our case here, $\delta\varphi_{\mathbf{k}}$ can be seen as two-fermion spinless wavefunction while $\delta\langle\phi\rangle$ gives an one-meson physics of the system in this regime. Furthermore, \mathbf{k} stands for the relative momentum of two fermions and its total momentum is constant of motion for this context of uniform system, therefore there is no explicit dynamics involved. Notice also that for scalar sector, the particles do not depend on the momentum in this context.

III. RPA EQUATIONS AS A SCATTERING PROBLEM

In the last section, we obtained the linear approximation for the gaussian equations of motion. They describe the physics of elementary excitations of the system. We shall show in this and next section that this coupled linear oscillator equations can be solved analytically giving the wavefunctions and the spectrum of excitations.

Let us first Fourier transform the wavefunctions to the energy representation,

$$\delta\varphi_{\mathbf{k}}(t) = \int d\omega \delta\varphi_{\mathbf{k}}(\omega) e^{i\omega t} \quad (3.1)$$

$$\delta\langle\phi\rangle(t) = \int d\omega \delta\langle\phi\rangle(\omega) e^{i\omega t}$$

where $\delta\varphi_{\mathbf{k}}$ and $\delta\langle\phi\rangle$ are now energy-dependent amplitudes. Substituting now (3.1) into (2.16)-(2.17) we have

$$(\omega^2 - 4\bar{\omega}^2) \delta\varphi_{\mathbf{k}}(\omega) = 2g|\mathbf{k}|\delta\langle\phi\rangle(\omega) \quad (3.2)$$

$$(\omega^2 - \mu^2 + Z\omega^2 - \Sigma) \delta\langle\phi\rangle(\omega) = -16\pi g \int_{\mathbf{k}} h(\mathbf{k}) \delta\varphi_{\mathbf{k}}(\omega) \quad (3.3)$$

Since the oscillation amplitudes play the roles of wavefunctions of quantum particles, it is then more convenient to treat this system as a coupled channel scattering problem with appropriate boundary conditions [3].

Next, we will discuss the scattering process where the source is a two-fermion wave. In this case, we can solve (3.3) as follows:

$$\delta\langle\phi\rangle(\omega) = \frac{-16\pi g}{\omega^2 - \mu^2 + Z\omega^2 - \Sigma + i\eta} \int_{\mathbf{k}'} h(\mathbf{k}') \delta\varphi_{\mathbf{k}'}(\omega) . \quad (3.4)$$

In this expression we have included the boundary condition of there is no incident wave of $\delta\langle\phi\rangle$. Substituting this into (3.2) and rewriting it in terms of a new variable

$$\Psi_{\mathbf{k}} = \sqrt{\bar{k}_0} \delta\varphi_{\mathbf{k}} \quad (3.5)$$

one finds

$$(\omega^2 - 4\bar{k}_0^2) \Psi_{\mathbf{k}} = \frac{-32\pi g^2}{\omega^2 - \mu^2 + Z\omega^2 - \Sigma + i\eta} \frac{|\mathbf{k}|}{\sqrt{\bar{k}_0}} \int_{\mathbf{k}'} \frac{|\mathbf{k}'|}{\sqrt{\bar{k}_0}} \Psi_{\mathbf{k}'} , \quad (3.6)$$

where the Green's Function includes the effects of coupling of Ψ to $\delta\langle\phi\rangle$. Notice also that the potential is separable in the sense that [8]

$$\langle\mathbf{k}|V|\mathbf{k}'\rangle = v(\mathbf{k})v(\mathbf{k}') = \frac{|\mathbf{k}|}{\sqrt{\bar{k}_0}} \frac{|\mathbf{k}'|}{\sqrt{\bar{k}_0}} \quad (3.7)$$

It is now convenient to rewrite it as an integral equations,

$$\begin{aligned} \Psi(\mathbf{k}, \mathbf{q}; \omega) &= \alpha \delta(\mathbf{q} - \mathbf{k}) \\ &+ \frac{1}{[\omega^2 - 4\bar{k}_0^2 + i\epsilon]} \frac{-32\pi g^2}{[\omega^2 - \mu^2 + Z\omega^2 - \Sigma + i\eta]} v(\mathbf{k}) \int_{\mathbf{k}'} v(\mathbf{k}') \Psi(\mathbf{k}', \mathbf{q}; \omega) , \end{aligned} \quad (3.8)$$

where \mathbf{q} is the relative momentum for two incident quasi-fermions and α is an overall phase factor. We have used the boundary condition of the outgoing wave condition ($+i\epsilon$) as solution of Eq.(3.7), but we could have chosen e.g. the incoming wave condition ($-i\epsilon$) or Van Kampen wave condition [9] or other conditions.

The integral equation (3.8) can be solved as usual. We first multiply the expression In order to solve we integrate this expression with respect to \mathbf{k} ,

$$\int_{\mathbf{k}} v(\mathbf{k}) \Psi(\mathbf{k}, \mathbf{q}; \omega) = \frac{\alpha v(\mathbf{q})}{1 + \left(\frac{32\pi g^2}{\omega^2 - \mu^2 + Z\omega^2 - \Sigma + i\epsilon} \right) I^+(\omega)} \quad (3.9)$$

where

$$I^+(\omega) = \int_{\mathbf{k}} \frac{|\mathbf{k}|^2}{\sqrt{\mathbf{k}^2 + \bar{m}^2}(\omega^2 - 4\bar{k}_0^2 + i\epsilon)} \quad (3.10)$$

Substituting this back result into (3.8) yields

$$\Psi(\mathbf{k}, \mathbf{q}; \omega) = \alpha \delta(\mathbf{q} - \mathbf{k}) + \frac{1}{\omega^2 - 4\bar{k}_0^2 + i\epsilon} \frac{\alpha v(\mathbf{q})}{\Delta^+(\omega)} \quad (3.11)$$

with

$$\Delta^+(\omega) = -\frac{1}{32\pi g^2} (\omega^2 - \mu^2 + Z\omega^2 - \Sigma) - I^+(\omega) \quad (3.12)$$

We have, thus, found an analytical solution for the elastic channel of two-fermion scattering problem given by (3.2)-(3.3).

The special form of interacting potential in this case allows one also to get easily a closed expression for the scattering matrix

$$T(\mathbf{k}, \mathbf{k}'; \omega) = v(\mathbf{k}) \frac{1}{\Delta^+(\omega)} v(\mathbf{k}') \quad (3.13)$$

with $\Delta^+(\omega)$ given by (3.12). In summary, this section has discussed the solutions for RPA equations. These elementary excitations describe a coupled channel scattering problem and we have studied the particular case of two-fermion elastic process. For this simple interacting

potential, it is easy to obtain closed expression for the two-fermion wavefunction and the scattering matrix, where several dynamical behavior can be read off from $\Delta^+(\omega)$. The remained problem is the divergent integral $I^+(\omega)$ in (3.12) which must be removed with the help of counterterms. We shall show in the next section that besides the counterterms used static discussion also apply we will need a convenient choice of Z .

IV. RENORMALIZATION AND BOUND STATE SOLUTION

This section will investigate the conditions for the existence of bound states of Dirac spin-1/2 particles in a system of scalar plasma, utilizing the framework developed in the previous sections [10]. In this context, the standard procedure is through an analysis of the positions of poles of the scattering matrix. The equation (3.12), however, has a divergent integral. We shall show next that the divergent terms can be kept directly under control with the help of (2.1)-(2.4) and a convenient choice of Z , yielding a finite expression for $\Delta^+(\omega)$.

Let us thus substitute the counterterms (2.1)-(2.4) into (3.12), after some algebra one gets

$$\Delta^+(\omega) = \frac{1}{8\pi g^2} \left[(1 + 4\pi g^2 L(m))\omega^2 - \mu^2 + 16\pi g^2 G(0) - 24\pi g^2 M^2 L(m) \right] - I^+(\omega) \quad (4.1)$$

with $I^+(\omega)$ given by (3.10). Notice that in the interval of $0 \leq \omega \leq 2M$ the integral I_ω is well defined and we can take $\epsilon = 0$. For $\omega > 2\bar{m}$, on the other hand, the spectrum is continuum. The calculation is straightforward and we find:

$$I(\omega) = Q - \frac{1}{8\pi^2} F(\omega) - \theta(\omega - 4\bar{m}^2) \frac{i}{8\pi} \left[\omega^2 - 4\bar{m}^2 \right], \quad (4.2)$$

where

$$Q = \frac{1}{4\pi} \left[\Lambda^2 + \left(\frac{\omega^2}{2} - 3\bar{m}^2 \right) \log \frac{2\Lambda}{m} \right] \quad (4.3)$$

and the finite terms are

$$F(\omega) = (\omega^2 - 6\bar{m}^2) \log \left(\frac{\bar{m}}{2m} \right) + \frac{2(4\bar{m}^2 - \omega^2)^{3/2}}{\omega} \tan^{-1} \sqrt{\frac{\omega^2}{4\bar{m}^2 - \omega^2}} \quad 0 \leq \omega^2 \leq 4\bar{m}^2$$

$$F(\omega) = (\bar{m}^2 - 6\omega^2) \log\left(\frac{\bar{m}}{2m}\right) + \frac{2(\omega^2 - 4\bar{m}^2)^{3/2}}{\omega} \log \frac{\omega + \sqrt{\omega^2 - 4\bar{m}^2}}{\omega - \sqrt{\omega^2 - 4\bar{m}^2}} \quad \omega^2 > 4\bar{m}^2 \quad (4.4)$$

$$(4.5)$$

Comparing (4.1) and (4.2) one notes immediately that there is still a logarithmic divergence left. This will be canceled with the following choice of wavefunction renormalization [4]

$$Z \equiv 4\pi g^2 L(m) . \quad (4.6)$$

The resulting finite expression is

$$\Delta^+(\omega) = -\frac{\pi\mu^2}{g^2 m^2} + F(\omega) - \theta(\omega - 4\bar{m}^2) \frac{i}{8\pi} [\omega^2 - 4\bar{m}^2] . \quad (4.7)$$

Discussion of the problem consist now in solving the equation

$$\Delta(\omega) = 0. \quad (4.8)$$

Depending on the value of ω the system has different dynamical behavior. When $\omega^2 < 0$ the system is unstable and for $\omega^2 > 0$ we are in the scattering regime. Solution of interest here is in the interval of $0 < \omega^2 < 4\bar{m}^2$. In this case, the system may present a stable bound state if one finds ω_B such that $\Delta^+(\omega_B) = 0$. The fig. 1 illustrate $\Delta^+(\omega)$ for several combinations of μ and g in unit of m . Notice that for the same value of $g(=1)$, $\Delta^+(\omega)$ has a single(none) zero when $\mu/m < 1.794(\mu/m > 1.794)$. A natural way to interpret this result is that the meson mass determines the range of the Yukawa potential for a same coupling. When μ of the system is large, it is more difficult to the fermions to interact and consequently decreases the probability of forming a bound state, and vice-versa. One can, however, compensate with increases of values of g . In order to see this, we have examine (4.7)-(4.8) for the parameter space μ/m versus g , the result is shown in the figure 2.

In summary, this work has investigated the physics of elementary excitations for the so called relativistic scalar plasma model. To reach this goal we have derived RPA equations for this sistem by linearizing the TDHFB equations obtained in a previous work. In this

context, the amplitudes of excitations are identified with quantum particles and the resulting equations allow one to study scattering processes nonperturbatively. We have solved this RPA equation analytically using well known procedures of scattering theory where the scattering amplitude obtained has a simple form. We have also shown that the usual definitions of counterterms can be applied to this resulting expression, from which relevant physics about the excitations of the system can be obtained. In particular, our results indicate existence of bound state for certain region of phase diagram.

Appendix A: Mean-field kinetic equations

In this appendix we will review briefly the results obtained of I. The method used there was developed earlier in the context the nonrelativistic nuclear many-body dynamics by Nemes and de Toledo Piza [11]. More recently, our group has applied this technique to ϕ^4 field theory [5] as well as to the Chiral Gross-Neveu model [6]. Our approximation focus on the time evolution of a selected set of observables, which in the case of scalar plasma system are the expectation values of linear, $\phi(x)$, and bilinear field operators such as $\phi(x)\phi(x)$, $\bar{\psi}(x)\psi(x)$, $\psi(x)\psi(x)$ and etc. Because of further convenience, we work instead with expressions which are bilinear in the creation and annihilation parts of the fields.

In the Heisenberg picture, $\phi(x)$ are scalar spin-0 fields

$$\phi(\mathbf{x}, t) = \sum_{\mathbf{p}} \frac{1}{(2Vp_0)^{1/2}} \left[b_{\mathbf{p}}(t)e^{i\mathbf{p}\cdot\mathbf{x}} + b_{\mathbf{p}}^\dagger(t)e^{-i\mathbf{p}\cdot\mathbf{x}} \right] , \quad (\text{A.1})$$

where $b_{\mathbf{p}}^\dagger(t)$ and $b_{\mathbf{p}}(t)$ are boson creation and annihilation operators. For the spin-1/2 fields we have

$$\begin{aligned} \psi(\mathbf{x}, t) &= \sum_{\mathbf{k}, s} \left(\frac{M}{k_0} \right)^{1/2} \frac{1}{\sqrt{V}} \left[u_1(\mathbf{k}, s) a_{\mathbf{k}, s}^{(1)}(t) e^{i\mathbf{k}\cdot\mathbf{x}} + u_2(\mathbf{k}, s) a_{\mathbf{k}, s}^{(2)\dagger}(t) e^{-i\mathbf{k}\cdot\mathbf{x}} \right] , \\ \bar{\psi}(\mathbf{x}, t) &= \sum_{\mathbf{k}, s} \left(\frac{M}{k_0} \right)^{1/2} \frac{1}{\sqrt{V}} \left[\bar{u}_1(\mathbf{k}, s) a_{\mathbf{k}, s}^{(1)\dagger}(t) e^{-i\mathbf{k}\cdot\mathbf{x}} + \bar{u}_2(\mathbf{k}, s) a_{\mathbf{k}, s}^{(2)}(t) e^{i\mathbf{k}\cdot\mathbf{x}} \right] , \end{aligned} \quad (\text{A.2})$$

where $a_{\mathbf{k}, s}^{(1)\dagger}(t)$ and $a_{\mathbf{k}, s}^{(1)}(t)$ [$a_{\mathbf{k}, s}^{(2)\dagger}(t)$ and $a_{\mathbf{k}, s}^{(2)}(t)$] are fermion creation and annihilation operators associated with positive [negative]-energy solutions $u_1(\mathbf{k}, s)$ [$u_2(\mathbf{k}, s)$] of Dirac's equation.

In order to deal with condensate and pairing dynamics of the scalar field we define the Bogoliubov transformation as follows [12]:

$$\begin{bmatrix} d_{\mathbf{p}}(t) \\ d_{-\mathbf{p}}^\dagger(t) \end{bmatrix} = \begin{bmatrix} \cosh \kappa_{\mathbf{p}} + i\frac{\eta_{\mathbf{p}}}{2} & -\sinh \kappa_{\mathbf{p}} + i\frac{\eta_{\mathbf{p}}}{2} \\ -\sinh \kappa_{\mathbf{p}} - i\frac{\eta_{\mathbf{p}}}{2} & \cosh \kappa_{\mathbf{p}} - i\frac{\eta_{\mathbf{p}}}{2} \end{bmatrix} \begin{bmatrix} \beta_{\mathbf{p}}(t) \\ \beta_{-\mathbf{p}}^\dagger(t) \end{bmatrix}. \quad (\text{A.3})$$

where $d_{\mathbf{p}}$ is the shift boson operator defined by

$$d_{\mathbf{p}}(t) \equiv b_{\mathbf{p}}(t) - B(t)\delta_{\mathbf{p},0} \quad \text{with} \quad B_{\mathbf{p}}(t) \equiv \langle b_{\mathbf{p}}(t) \rangle = Tr_{\text{BF}} [b_{\mathbf{p}}(t)\mathcal{F}] \quad (\text{A.4})$$

In the case of fermions we restrict ourselves for simplicity to Nambu transformation, which can be parametrized in a form that incorporates the unitarity constraints as

$$\begin{bmatrix} a_{\mathbf{k},s}^{(1)} \\ a_{\mathbf{k},s}^{(2)} \\ a_{-\mathbf{k},s}^{(1)\dagger} \\ a_{-\mathbf{k},s}^{(2)\dagger} \end{bmatrix} = \begin{bmatrix} \cos \varphi_{\mathbf{k}} & 0 & 0 & -e^{-i\gamma_{\mathbf{k}}} \sin \varphi_{\mathbf{k}} \\ 0 & \cos \varphi_{\mathbf{k}} & e^{-i\gamma_{\mathbf{k}}} \sin \varphi_{\mathbf{k}} & 0 \\ 0 & -e^{i\gamma_{\mathbf{k}}} \sin \varphi_{\mathbf{k}} & \cos \varphi_{\mathbf{k}} & 0 \\ e^{i\gamma_{\mathbf{k}}} \sin \varphi_{\mathbf{k}} & 0 & 0 & \cos \varphi_{\mathbf{k}} \end{bmatrix} \begin{bmatrix} \alpha_{\mathbf{k},s}^{(1)} \\ \alpha_{\mathbf{k},s}^{(2)} \\ \alpha_{-\mathbf{k},s}^{(1)\dagger} \\ \alpha_{-\mathbf{k},s}^{(2)\dagger} \end{bmatrix} \quad (\text{A.5})$$

The next step is to obtain the mean-field time evolution for the mean values of the gaussian observables in the context of the initial-value problem. In other words, we want the gaussian mean-field equations of motion for the parameters $\varphi_{\mathbf{k}}(t)$, $\gamma_{\mathbf{k}}(t)$, $\eta_{\mathbf{p}}(t)$, $\kappa_{\mathbf{p}}(t)$, for the quasi-particle occupation numbers $\nu_{\mathbf{k},\lambda}(t)$, $\nu_{\mathbf{p}}(t)$ and for $\langle \phi \rangle$ and $\langle \Pi \rangle$. In Ref.[1] we obtained for the bosonic variables

$$\dot{\nu}_{\mathbf{p}} = Tr_{\text{BF}}[\beta_{\mathbf{p}}^\dagger \beta_{\mathbf{p}}, H]\mathcal{F} \quad (\text{A.6})$$

$$-i\dot{\kappa}_{\mathbf{p}} - e^{-\kappa_{\mathbf{p}}}(\dot{\eta}_{\mathbf{p}} + \dot{\kappa}_{\mathbf{p}}\eta_{\mathbf{p}}) = \frac{Tr[\beta_{\mathbf{p}}^\dagger \beta_{-\mathbf{p}}^\dagger, H]\mathcal{F}}{1 + 2\nu_{\mathbf{p}}} \quad (\text{A.7})$$

$$i\langle \dot{\phi}(t) \rangle = Tr_{\text{BF}}[\phi(t), H]\mathcal{F} \quad (\text{A.8})$$

In the case of fermionic variables, we obtained

$$\dot{\nu}_{\mathbf{k},\lambda} = Tr_{\text{BF}} \left([\alpha_{\mathbf{k},s}^{(\lambda)\dagger} \alpha_{\mathbf{k},s}^{(\lambda)}, H] \mathcal{F} \right) \quad (\text{A.9})$$

$$[i\dot{\varphi}_{\mathbf{k}} + \dot{\gamma}_{\mathbf{k}} \sin \varphi_{\mathbf{k}} \cos \varphi_{\mathbf{k}}] e^{-i\gamma_{\mathbf{k}}} = \frac{Tr_{\text{BF}} \left([\alpha_{-\mathbf{k},s}^{(1)} \alpha_{\mathbf{k},s}^{(2)}, H] \mathcal{F} \right)}{1 - \nu_{\mathbf{k},s}^{(1)} - \nu_{\mathbf{k},s}^{(2)}} . \quad (\text{A.10})$$

Our implementation of the gaussian mean-field approximation consists in approximating \mathcal{F} by a truncated many-body density operator $\mathcal{F}_0(t) = \mathcal{F}_0^{\text{B}} \mathcal{F}_0^{\text{F}}$. The factorized form of the $\mathcal{F}_0(t)$ embodies what we refer to as the double mean field approximation. The subsystem densities \mathcal{F}_0^{B} and \mathcal{F}_0^{F} are in fact unit trace gaussian densities, written in the form of an exponential of a bilinear, Hermitian expression in the creation and annihilation parts of the bosonic and of the fermionic fields, respectively. They can be, however, rewritten in diagonal form, (1.4) and (1.5), when one uses Bogolyubov quasiboson and quasifermion operators. With these assumptions the equations (A.6)-(A.10) are closed now and they will determinate the time rate of change of gaussian variables. The results are shown in (1.6)-(1.9).

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References

- [1] E.R. Takano Natti, C-Y. Lin, A.F.R. de Toledo Piza and P.L. Natti, “Initial-Value Problem in Quantum Field Theory: an Application to the Relativistic Scalar Plasma”, hep-th/9805060.
- [2] A. Kerman and C-Y. Lin, Ann. Phys.(N.Y.) **241**, 185 (1995).
- [3] A. Kerman and C-Y. Lin, Ann. Phys.(N.Y.) **269**, 55 (1998).
- [4] J. D. Alonso and R. Hakim, Phys. Rev. **D 38**, 1780 (1988);

- [5] L. C. Yong and A. F. R. de Toledo Piza, Phys. Rev. **D 46**, 74 2 (1992); L. C. Yong, Doctoral Thesis, University of São Paulo, 1991 (unpublished).
- [6] P. L. Natti and A. F. R. de Toledo Piza, Phys. Rev. **D54**, 1 (1996); P. L. Natti, Doctoral Thesis, University of São Paulo, 1995 (unpublished).
- [7] A. K. Kerman and S. E. Koonin, *Ann. Phys. (N.Y.)***100**, 332 (1976).
- [8] R. Newton, *Scattering Theory of Waves and Particles*, Springer-Verlag 1982
- [9] M.C. Nemes, A.F.R. de Toledo Piza, and J. da Providência, Physica **146A**, 282 (1987).
- [10] P. L. Natti and A. F. R. de Toledo Piza, Phys. Rev. **D 55**, 3403 (1997).
- [11] A. F. R. de Toledo Piza, in *Time-Dependent Hartree-Fock and Beyond*, edited by K. Goeke and P.-G. Reinhardt, Lectures Notes in Physics 171 (Springer-Verlag, Berlin, 1982); M. C. Nemes and A. F. R. de Toledo Piza, Phys. Rev. **C 27**, 862 (1983); B. V. Carlson, M. C. Nemes and A. F. R. de Toledo Piza, Nucl. Phys **A 457**, 261 (1986); M. C. Nemes and A. F. R. de Toledo Piza, Physica **A 137**, 367 (1986).
- [12] P. Ring and P. Schuck, *The Nuclear Many-Body Problem*, Spring-Verlag, New York (1980).

Figure Captions

Figure 1. The behavior of the function $\Delta(\omega)$ as a function of energy in unit of m .

Figure 2. Existence of bound state of two fermion as a function of parameters μ/m and g .